Term 4 Assignment - Fractal Triangles

An outstanding report

Introduction

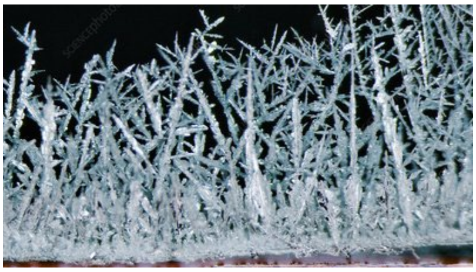
Fractals - Fractals are one of the most complex things in our universe. It shows an unending number of repeating patterns of a thing that is similar in different scales. Fractals are formed by repeating a process over and over and again for an infinite number of times. Fractals are so complex that mathematical considers it as a feature of chaos.

Different Types of Fractals in nature

Fractals are seen all over the universe and around us.

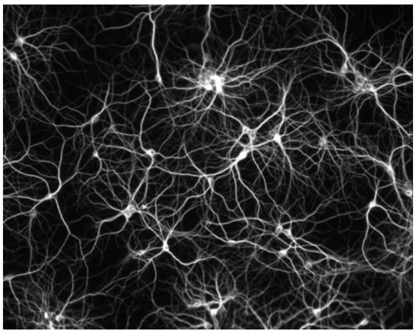
Chemistry - Fractals are common in chemistry, especially in crystallisations. You can see the crystals and how it represents how fractals can look like.

Crystallising copper

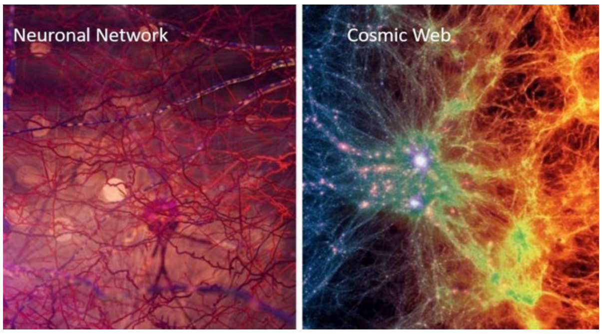


Crystallising silver

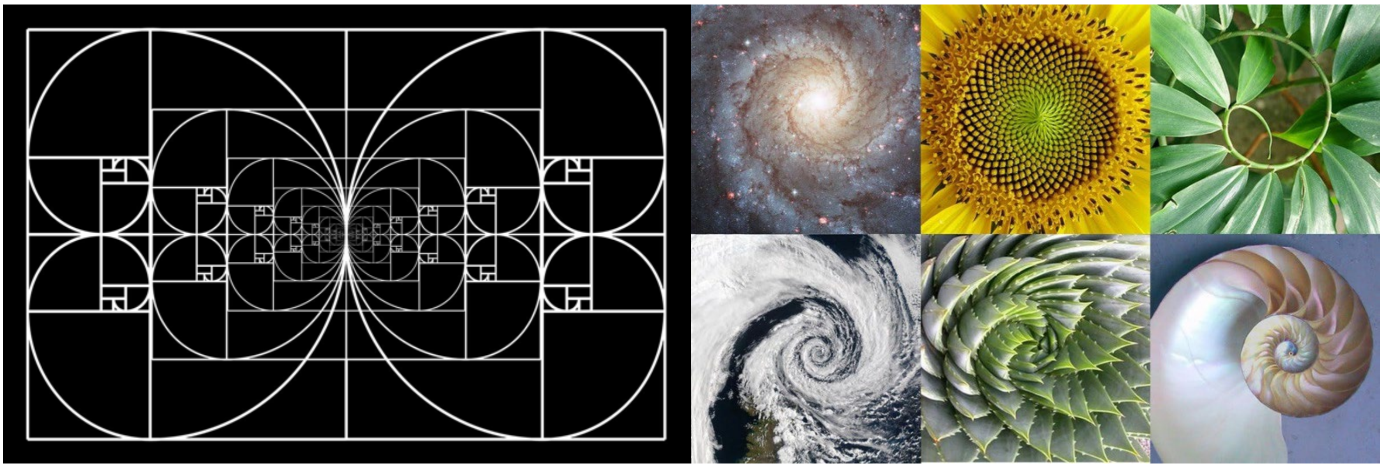
Our brain and the universe - Our brains are one of the most complex things in the existence. So is the universe. You can see the similarities and complexities of the brain and the universe, and they totally show how fractals look like becausee fractals are also complex just like them.



Neurone structures in the brain



The Golden ratio - The golden ratio is an irrational number and is indeed a fractal like sequence. It is also an infinitely complex shape and is also known to be all over nature. Both the golden ratio and fractals can be seen in nature, so it shows how the golden ratio is related to fractals.



The Sierpinski Triangle - The sierpinski triangle is one of the most famous fractals in mathematics. It was named after a Polish mathematician, Waclaw Sierpinski. The Sierpinski triangle is an equilateral triangle that has infinitely more similar triangles inside itself that are also equilateral triangles. It can be done by connecting the exact mid points of an equilateral triangle which produces 4 triangles. One right side down triangle and 3 right side up triangles and they are all congruent to each other. You can repeat this process for any right side up triangle that is formed for an infinite number of times. This makes it a fractal because the original triangle is showing itself in similar scales inside itself for an infinite number of times.

Usage of fractals in real life - Fractals are mainly used by humans to predict patterns such as: -
fractals cities: some cities grow in a fractal like pattern, and you can use fractals to figure out the growth of the population.
- fractal medicine: scientists can use fractals to study how bacteria and cells grow because they grow in a fractal like pattern.
- fractal geography: you can use fractals to predict things in geography such as mountain formations and river flowing structures.

The Sierpinski Triangle

Iteration	Number of right side up triangles	Number of right side down triangles	Index number
0	1	0	3^0
1	3	1	3^1
2	9	3	3^2
3	27	9	3^3
4	81	27	3^4
5	243	81	3^5
6	729	243	3^6

<u>How many triangles have the class drawn?</u>

- Amount to students in class = 16 students
- Lest assume everyone did at least one sierpinski triangle and did at least 4 iterations
- Overall right side up triangles in 4 iterations = 3+9+27+81= 120
- Overall right side down triangles in 4 iterations= 1+3+9+27= 40

Equation- $(right\ side\ up\ triangles + \ right\ side\ down\ triangles)16$

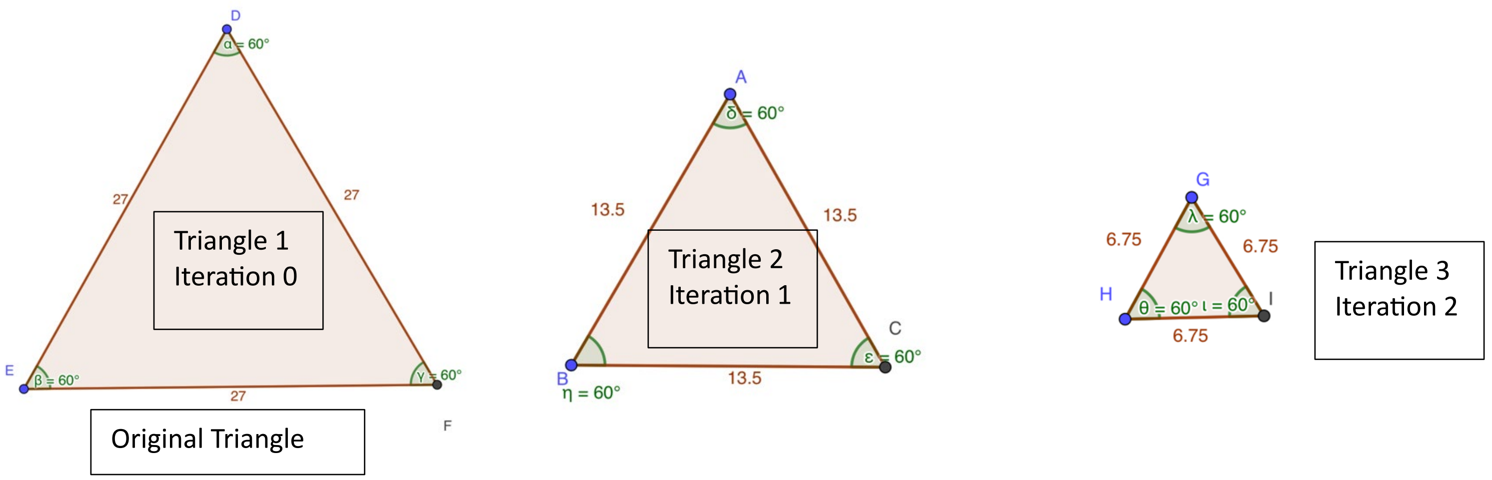
$$(120 + 40)16$$

$$160 * 16 = 2560$$

The class would have drawn approximately 2560 triangles.

Calculating similarity

<u>Equilateral Sierpinski Triangle</u>



Every single Equilateral triangle will always have 3 angles that are 60° so basically, we can assume that it makes all of the triangles in the Sierpinski triangle similar to each other. (AAA)

You can also prove it another way by using scale factor because all the triangles should be similar in scale because they are equilateral triangles.

Scale factor between triangle DEF and ABC

SF = $\frac{Image}{original}$

SF = $\frac{13.5}{27} = \frac{1}{2}$

Scale factor between triangle ABC and GHI

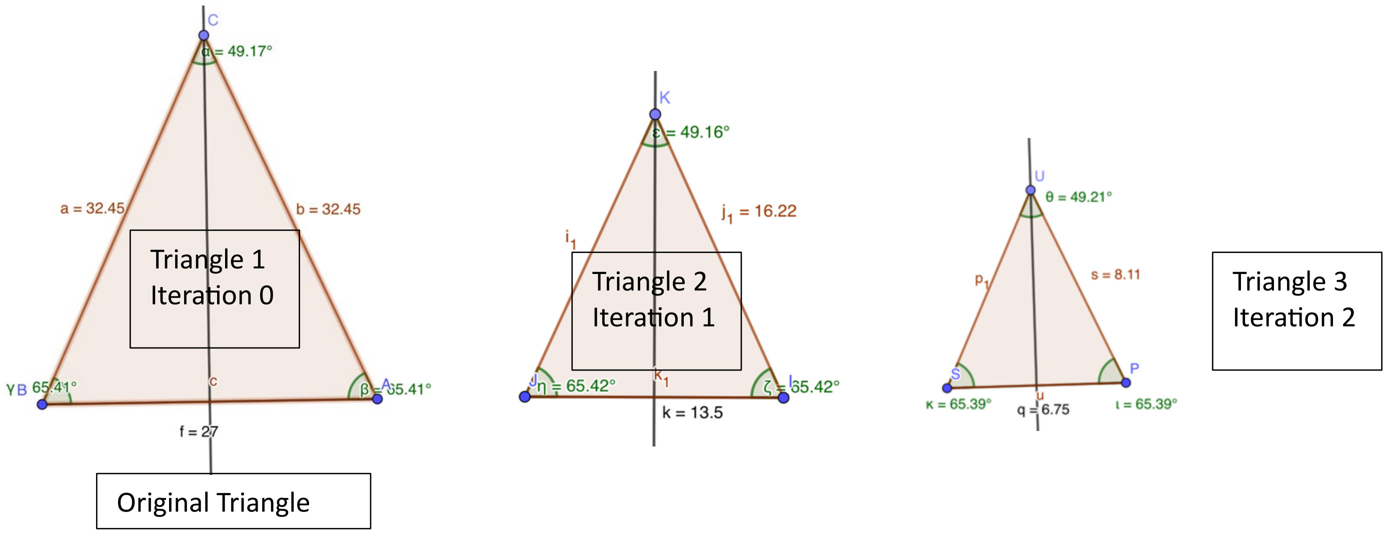
SF = $\frac{6.75}{13.5} = \frac{1}{2}$

Scale factor between Triangle DEF and GHI SF= $6.75/27$ SF= $1/4$

We can say that the scale of a triangle drawn in the sierpinski triangle is halved each iteration. We can also see the that the sides are also halved exactly as well. Lastly, we can also see that the sides are similar in scale as well so therefore, we can say that-

Triangle DEF ~ Triangle ABC (SSS)
Triangle ABC ~ Triangle GHI (SSS)
Triangle DEF ~ Triangle GHI (SSS)

<u>Isosceles Sierpinski Triangle</u>



You can also prove similarity of these isosceles triangles with scale factors.
SF between Triangle ABC and IJK

SF = $\frac{13.5}{27}$ or $\frac{16.22}{32.45}$

SF = $\frac{1}{2}$

SF between Triangle IJK and PSU

SF= $\frac{6.75}{13.5}$ or $\frac{8.11}{16.22}$

SF= $\frac{1}{2}$

SF between Triangle ABC and PSU

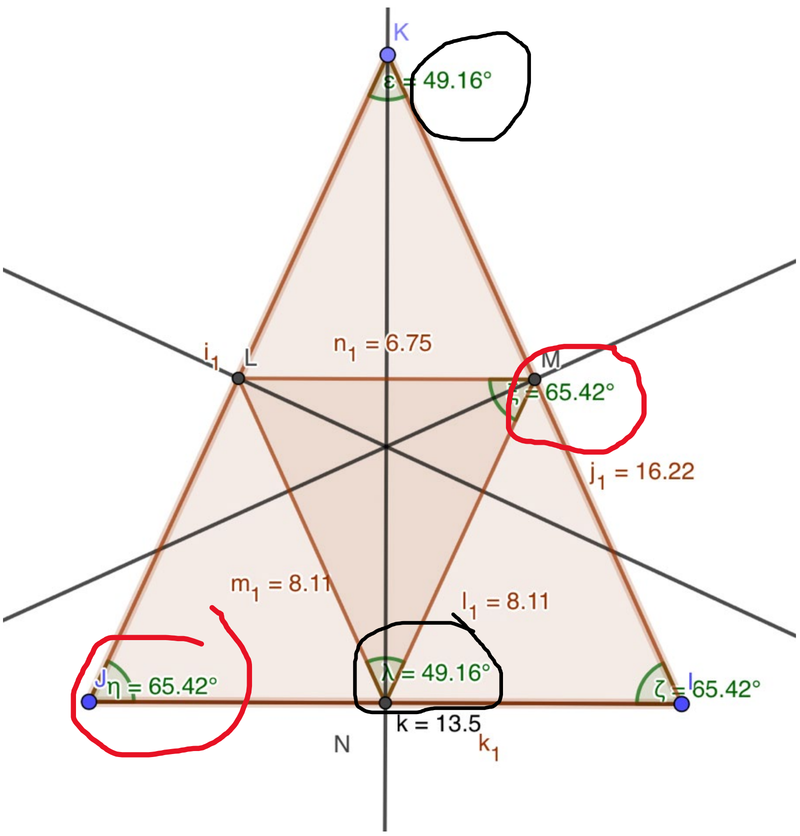
SF=$\frac{6.75}{27}$ or $\frac{8.11}{32.45}$

SF= $\frac{1}{4}$

Therefore -

Triangle ABC ~ Triangle IJK (SSS)
Triangle IJK ~ Triangle SUP (SSS)
Triangle SUP ~ Triangle ABC (SSS)

The three isosceles triangles above show angles that are quite similar but still slightly different by decimal points. I think this is just a drawing error because the angle should've been the same for each triangle.



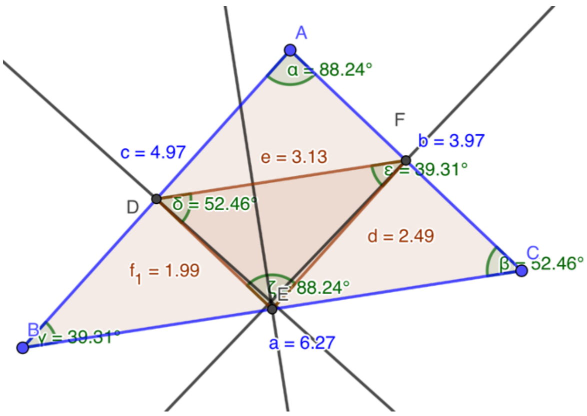
As you can see in this picture, if it was drawn inside an Isosceles triangle correctly, the angles would've been the same for each one.

So technically we can say that-

Angle JKI = Angle MNL
Angle IJK = Angle LMN
Angle KIJ = Angle NLM

Therefore-
Triangle IJK ~ Triangle MNL (AAA)

A Random triangle



Angle ABC = Angle DFE
Angle ACB = Angle EDG
Angle BAC = Angle DEF

Therefore- Triangle ABC ~ Triangle DEF (AAA)

SF between Side AB and EF

SF= $\frac{2.49}{4.97} = Approximately\ \frac{1}{2}$

SF between Side BC and DF

SF = $\frac{3.13}{6.27} = Approximately\ \frac{1}{2}$

SF between Side AC and DE

SF= $\frac{1.99}{3.97} = Approximately\ \frac{1}{2}$

Side AB ~ Side EF
Side BC ~ Side DF
Side AC ~ Side DE

Therefore-
Triangle ABC ~ Triangle DEF (SSS)

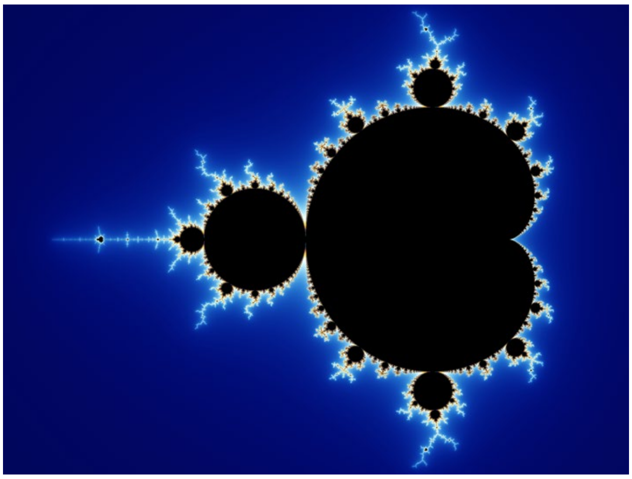
For the triangles, the scale factor seems to be approximately half instead of exactly half, but I think if the length of sides were whole numbers instead of decimals, there's a possibility that the scale factor might be exactly half.
As you can see here, these triangles are also similar to each other even though the length of the original triangle are completely random. That no matter what type of triangle it is, if it was to be drawn like a sierpinski triangle, all the triangles would be similar.

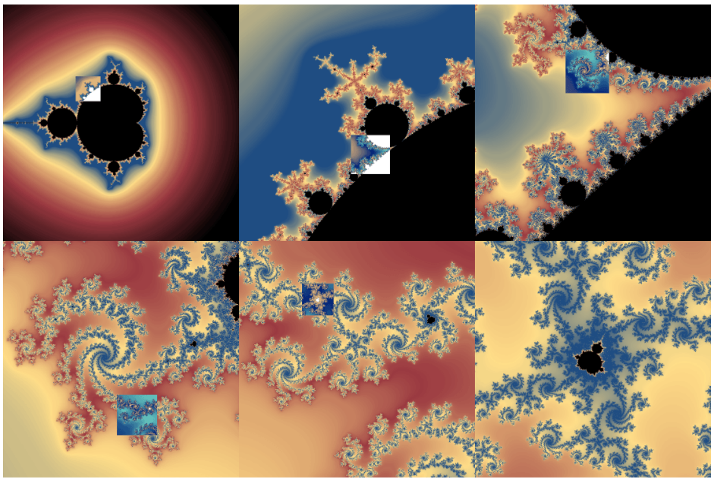
After looking at the equilateral, isosceles and the randomly structured triangle, I can say that if a smaller triangle was to be drawn inside an original triangle by the exact mid points of each side of the triangle, the triangle that has been drawn, plus a few more produced by that triangle will be similar to the original triangle. The other triangles produced by that drawing will also be congruent to each other.

Discussion

The triangles produced in the Sierpinski triangle multiply as 3^x and where x represents the number of iterations. This only shows the multiplication of right side up triangles so in order to find the upside-down triangles, you must take away 1 to each iteration becausee there will be 3 times less upside-down triangles per iteration. Eg. 3^{x-1}. If you want the number of triangles produced in a certain iteration, you simply add those two numbers together.
Drawing the Sierpinski triangles take a lot of time, but I've found one shortcut that made things easier. So, I know that all the right side up triangles are congruent and basically translates horizontally in an equal space. So, I measured the mid-point of the furthest side of a triangle on the left side and did the same for the right. Then I simply connected both sides with a ruler and drew lines in triangles where it was necessary. These shortcuts exist because the triangles are similar and also placed in the right positions. Even with the shortcuts, there were still limitations because humans can't physically draw perfectly. The lines might not be equal, the triangles might not be similar and even on a computer, if you draw it wrongly, there might be errors. Two sides of the triangle were also pixelated which makes it even harder to measure. There will be always some kind of limitations becausee not everything is perfect.

Fractals can be so complex sometimes that some mathematicians even consider it as how God's mind might work. One great example is the Mandelbrot set, discovered by Benoit Mandelbrot. It is the ultimate fractal and the complexity of it is out of this universe. This can't be found anywhere in the universe, and it was only discovered because of a programming on a computer. The Mandelbrot set doesn't look that complex until you zoom in on any point of the picture. Whichever point you zoom in, there will be always be a set of fractals waiting to be discovered. There are fractals on fractals on fractals and it goes on forever and there are even more Mandelbrot sets in the Mandelbrot set. This shows how complex God's creation are and how fractals show how his mind works.





Conclusion

In this assignment, I've found lots of new ideas and concepts about fractals and how we can identify them in nature and more importantly how to use them. With all my findings, the answer to the big question is Yes. All the triangles in a Sierpinski triangle are always similar. It doesn't matter what type of triangle it is; they will always be similar.

Reference List

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Tree hugger (n.d.). 9 Amazing Fractals Found in Nature. https://www.treehugger.com/amazing-fractals-found-in-nature-4868776