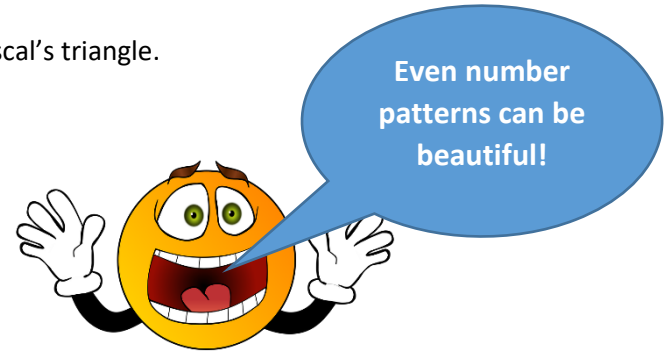
Pascal's Triangle - Number Pattern Investigation

Aim:

To identify the *beautiful* number patterns associated with Pascal's triangle.



Student response to this investigation:

"Wow – Pascal's triangle is like a cheat sheet for patterns"
Jake April 2023

Part A: Binomial Expansion

Expand and simplify the following

$(x+1)^0$

$(x+1)^1$

$(x+1)^2$

$(x+1)^3$

$(x+1)^4$

Part A: Summary

$(x+1)^0$	
$(x+1)^1$	
$(x+1)^2$	
$(x+1)^3$	
$(x+1)^4$	

Part B: Pascal's Triangle

1. Fill in the blank triangle handout
2. Comment on the **symmetry**
3. Number each row – the first row is "0"
4. **Binomial Expansion:** What do you observe as you look back at Part A?

5. **Add up each row. – Exponential growth 2^x**
   a. Write this next to each row.

   b. What do you notice?

   c. Write the simplified version next to each addition

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6. Powers of 11

a. Use a calculator to list the powers of 11

11^0

11^1

11^2

11^3

11^4

11^5

11^6

11^7

b. What do you notice?

c. Oops what happened at 5?

d. And 6?

(More help: https://www.mathsisfun.com/pascals-triangle.html)

Part C: Odds and Evens

7. Colour every odd number on a *new pre-filled triangle* (with at least 15 rows). Your teacher will hand you this.

8. What do you notice? **(Fractals)**

Part D: Diagonals

You will need a *new triangle* and 4 contrasting colours for this

9. **Triangular numbers** – explore with ''dots"
    a. Draw the first 5 triangular numbers as equilateral triangles

    b. Write the 'number sentence' next to each triangular number drawing

    c. What is a triangular number?

    d. Draw these first 5 triangular numbers again as right-angled triangles

10. If n is the number of dots on the base of the triangle – identify a rule to work out triangular numbers. (hint: think about area of rectangles)

11. Use your rule to list the next 3 triangular numbers

12. Triangular number summary

1	2	3	4	5	6	7	8

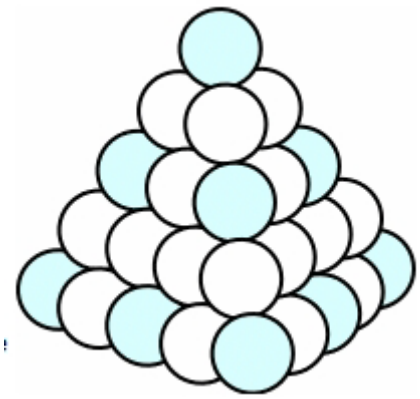
13. Colour all of the triangular numbers down the left side of the Pascals triangle in the same colour

14. Colour all of **the 1's** down the left side in another colour

15. Colour the **counting numbers** down the left side in another colour

16. **Tetrahedron Numbers** – stacking balls

(Triangular numbers in 3D)



a. Fill in the table

N (height)	Triangular number (number of balls in each layer)	**Tetrahedron Number** (Total balls)
1	1	1
2	3	4

b. Colour the Tetrahedron numbers in another colour

Part E: Square Numbers

17. Consider the square numbers and identify the pattern in the triangle
    a. Fill in the rest of the table

    b. On a new triangle – colour in such a way to highlight some examples

2^2	1 + 3	= 4
3^2	3 + 6	= 9
4^2	6 + 10	= 16

Part F: Fibonacci Numbers

Look up the Fibonacci Sequence of numbers and identify where they appear on the triangle

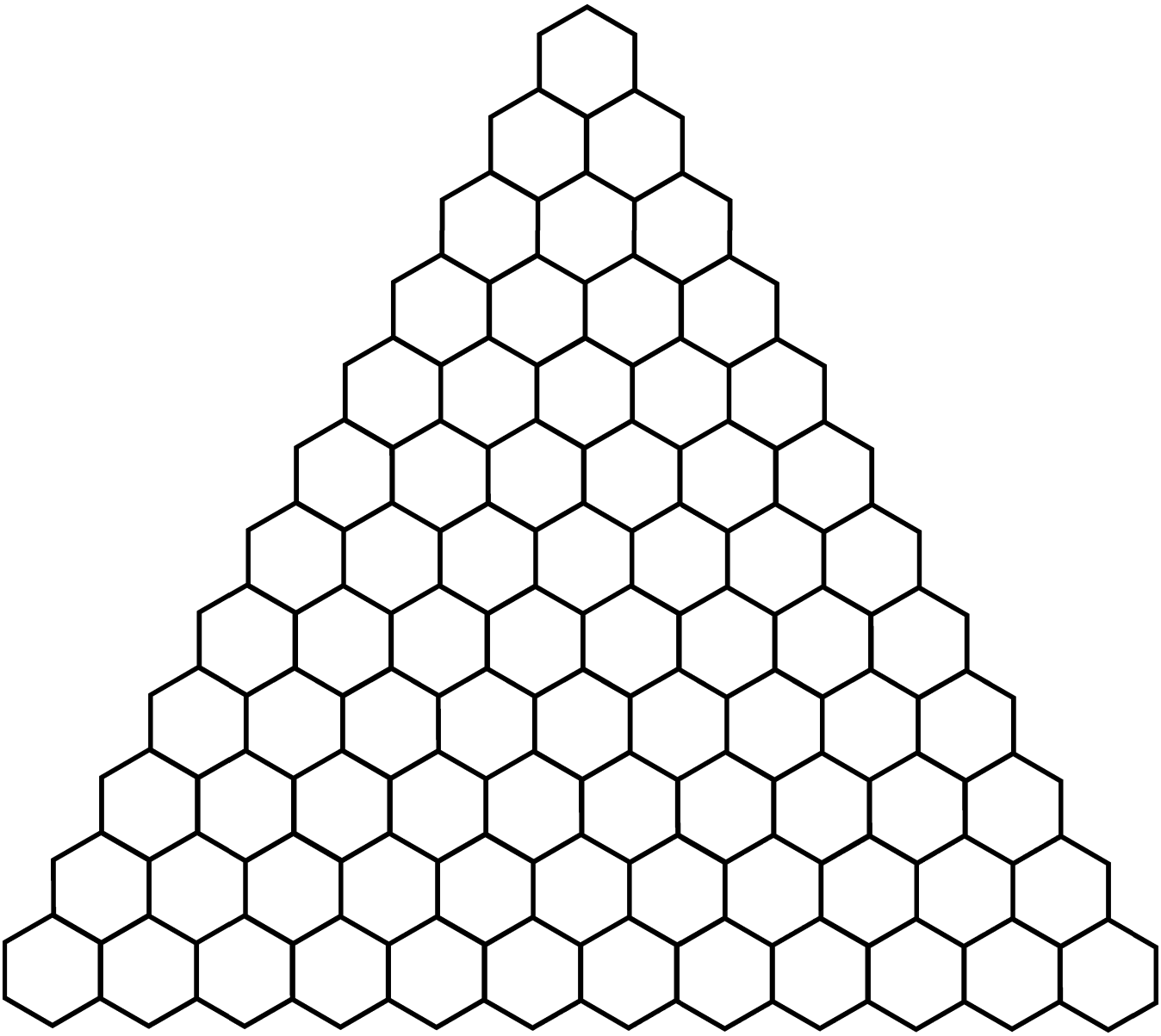
Part G: Probability

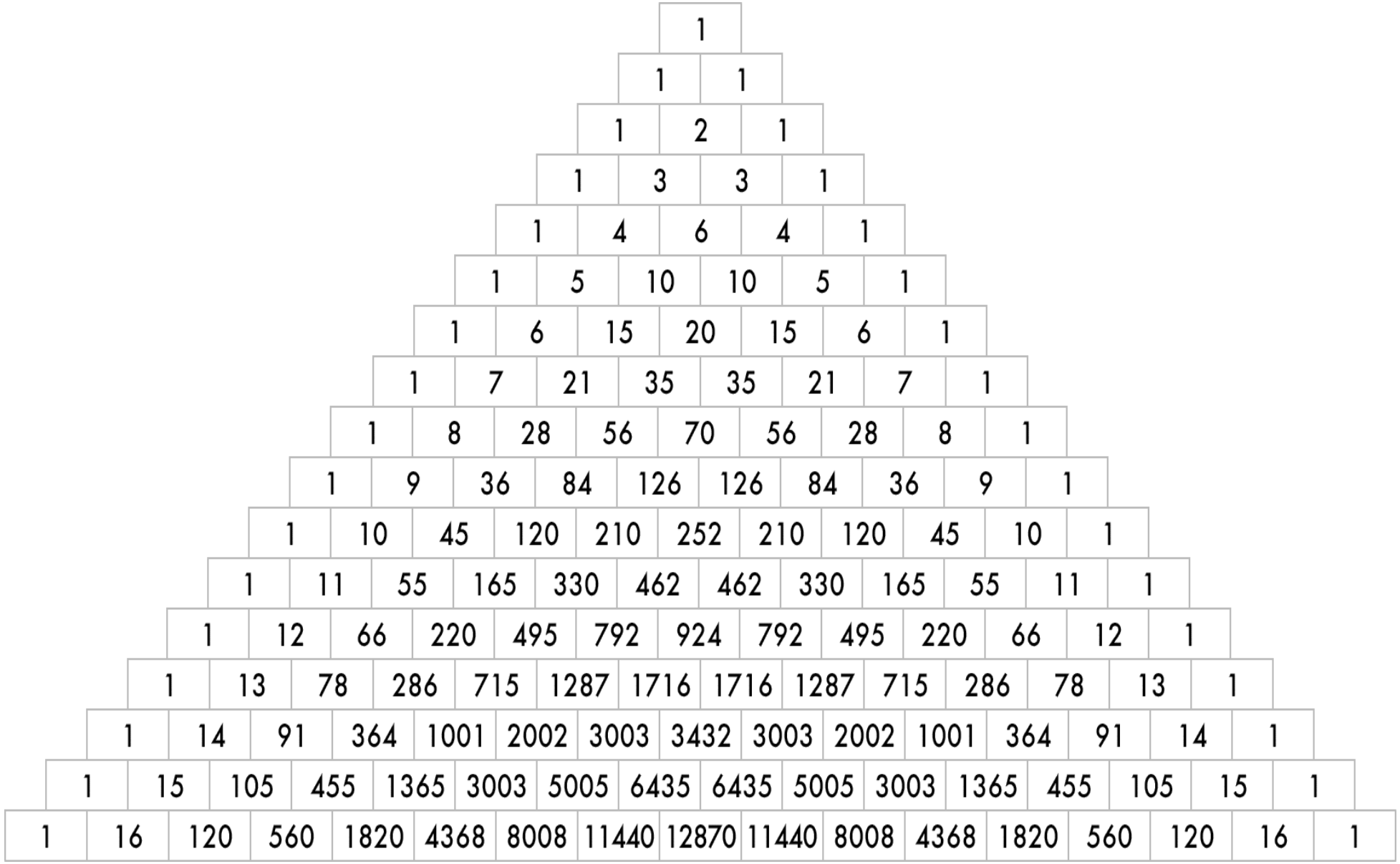
Toss a coin once; twice; three times; four times – list the number of outcomes (grouped in like groups)

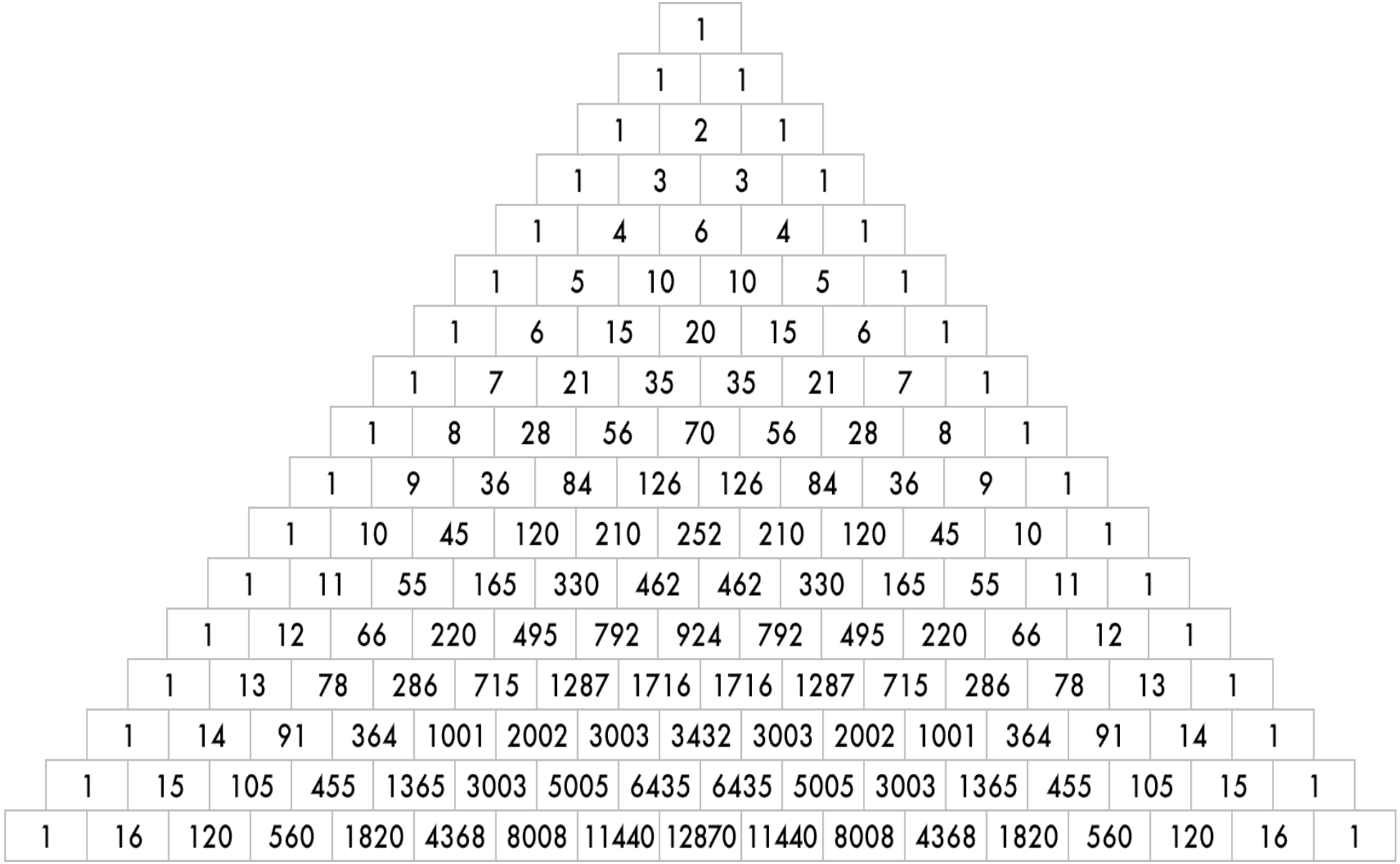
Number of tosses of a coin	Possible outcomes (grouped in like groups)	Pascal's triangle

Pascal's Triangle

Blank




1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
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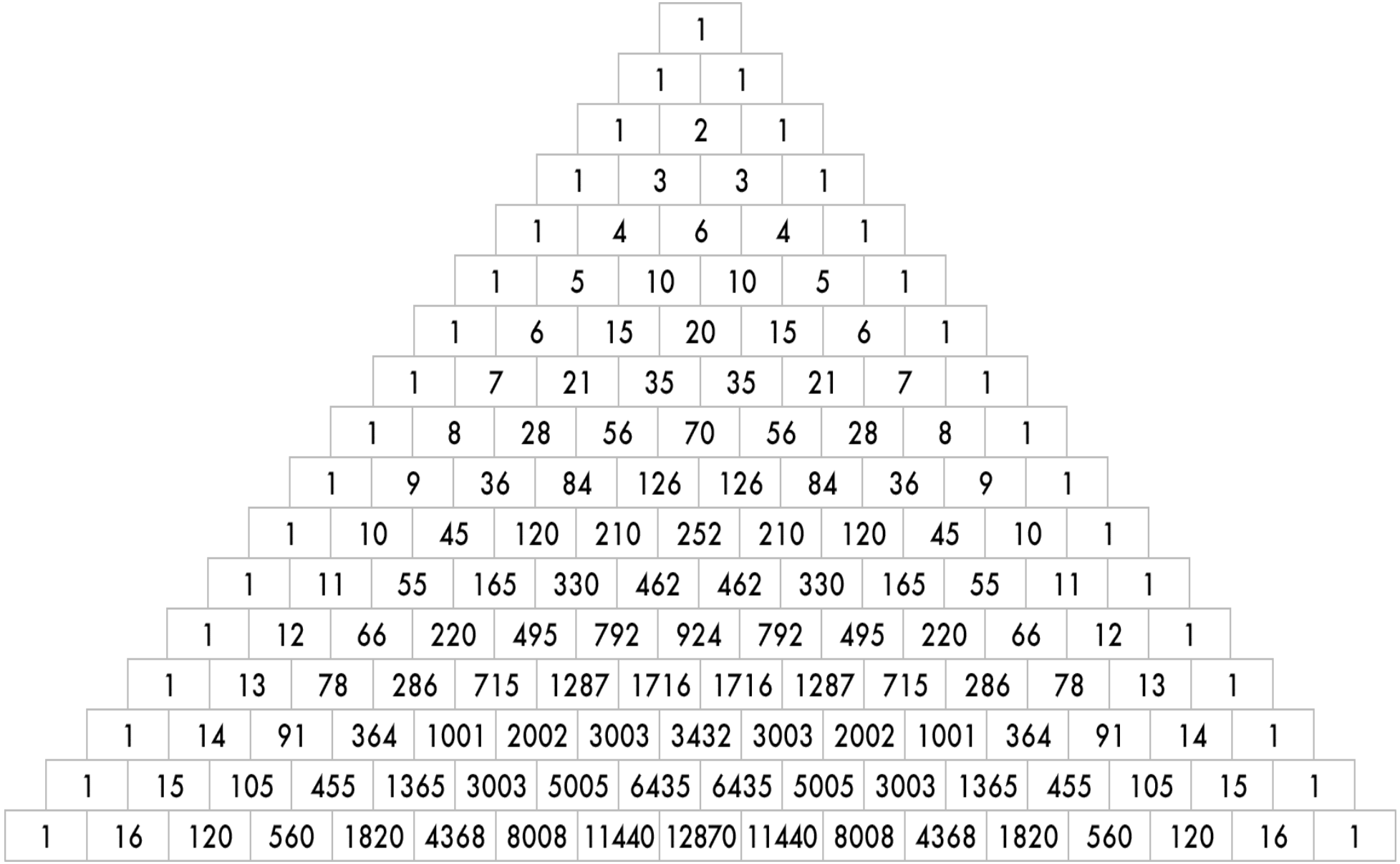
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
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1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1